Relazioni fondamentali

Valgono ancora le relazioni fondamentali

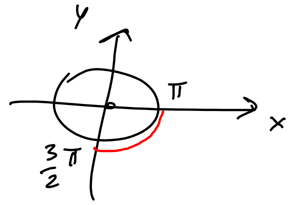
$$\sin^2\alpha + \cos^2\alpha = 1, \qquad \tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

già ricavate per angoli compresi tra 0 e $90°$, ma occorre fare attenzione ai segni, a seconda del quadrante in cui si trova l'angolo:

	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$
$\sin\alpha$	$\sin\alpha$	$\pm\sqrt{1-\cos^2\alpha}$	$\pm\dfrac{\tan\alpha}{\sqrt{1+\tan^2\alpha}}$
$\cos\alpha$	$\pm\sqrt{1-\sin^2\alpha}$	$\cos\alpha$	$\pm\dfrac{1}{\sqrt{1+\tan^2\alpha}}$
$\tan\alpha$	$\pm\dfrac{\sin\alpha}{\sqrt{1-\sin^2\alpha}}$	$\pm\dfrac{\sqrt{1-\cos^2\alpha}}{\cos\alpha}$	$\tan\alpha$

Per esempio:

$$\sin\alpha = -\frac{3}{5} \quad e \quad \frac{3}{2}\pi < \alpha < 2\pi$$



allora $\circ$ $\cos\alpha = +\sqrt{1-\sin^2\alpha} =$

$$= \sqrt{1-\frac{9}{25}} = \frac{4}{5} \qquad (\text{perché } \cos\alpha > 0)$$

$\circ$ $$\tan\alpha = \frac{-\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{4} \qquad \left(\text{nel IV quadrante } \tan\alpha < 0\right)$$

Esercizi

1) $$\sin^2\alpha + \cos^2\alpha + \tan^2\alpha = 1 + \tan^2\alpha = \frac{\cos^2\alpha + \sin^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha}$$

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2) $\left(1+\tan\alpha+\frac{1}{\cos\alpha}\right)\cdot\left(1+\frac{1}{\tan\alpha}-\frac{1}{\sin\alpha}\right)=$

$$=\left(\frac{\cos\alpha+\sin\alpha+1}{\cos\alpha}\right)\cdot\left(\frac{\sin\alpha+\cos\alpha-1}{\sin\alpha}\right)=$$

$$=\frac{(\cos\alpha+\sin\alpha)^2-1}{\sin\alpha\cos\alpha}=\frac{\cos^2\alpha+\sin^2\alpha+2\sin\alpha\cos\alpha-1}{\sin\alpha\cos\alpha}=$$

$$=\frac{\cancel{1}+2\sin\alpha\cos\alpha-\cancel{1}}{\sin\alpha\cos\alpha}=2$$

3) $1+\cos\alpha+\frac{\sin^2\alpha}{\cos\alpha}-\frac{1}{\cos\alpha}-\frac{1}{1+\tan^2\alpha}=$

$$=1+\cos\alpha+\frac{1-\cos^2\alpha}{\cos\alpha}-\frac{1}{\cos\alpha}-\cos^2\alpha=$$

$$=1+\cancel{\cos\alpha}+\cancel{\frac{1}{\cos\alpha}}-\cancel{\cos\alpha}-\cancel{\frac{1}{\cos\alpha}}-\cos^2\alpha=\sin^2\alpha$$